3. Instant Runoff Voting (IRV)

Warmup

Skip, Norm, and Jesse are running for president. There are 100 voters and their preferences are as follows:

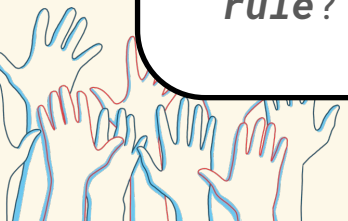
Number of votes

Rank	35	28	20	17
1	N	S	J	J
2	S	N	N	S
3	J	J	S	N

Q: If we just consider voters' first choice, did anyone win under ***majority rule***?

Q: If we just consider voters' first choice, did anyone win under ***plurality***?

Q: Who wins under the ***Borda count***?



Head-to-head contests

Definition

A ***head-to-head contest*** is a two-candidate election decided by majority rule.

Number of votes

Rank	35	28	20	17
1	N	S	J	J
2	S	N	N	S
3	J	J	S	N

In the example we just discussed, we can ask *how voters preferences would change if we forget about Jesse*.

Number of votes

Rank	35	28	20	17
1	N	S		
2	S	N	N	S
3			S	N

If Jesse is no longer in the picture, then for the block of 20 voters, Norm is their new first choice, and Skip is their new second choice. We can shift their preferences up then:

Number of votes

Rank	35	28	20	17
1	N	S	N	S
2	S	N	S	N

Head-to-head contests

By throwing out Jesse, we now have a ***head-to-head*** contest between Skip and Norm. This is now basically a majority rules election.

We see that Norm has 35+20=55 first place votes, and Skip has 28+17=45 first place votes. So we say that ***Norm wins a head-to-head election with Skip.***

Number of votes

Rank	35	28	20	17
1	N	S	N	S
2	S	N	S	N

Number of votes

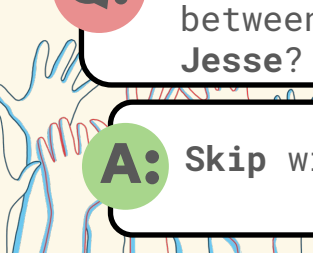
Rank	35	28	20	17
1	N	S	J	J
2	S	N	N	S
3	J	J	S	N

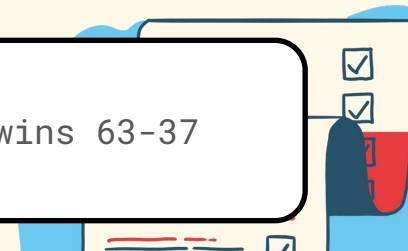
Q: Who wins in a head-to-head contest between **Skip** and **Jesse**?

A: **Skip** wins 63-37

Q: Who wins in a head-to-head contest between **Norm** and **Jesse**?

A: **Norm** wins 63-37





Condorcet winners and losers

So in head-to-head contests:

- Norm beats Jesse
- Skip beats Jesse
- Norm beats Skip

We notice two interesting things about this:

1. *Norm would **win** every head-to-head election he is involved in*
2. *Jesse would **lose** every head-to-head election she is involved in*

Candidates that satisfy these properties have names --- we call Norm a ***Condorcet winner*** and we call Jesse a ***Condorcet loser***.

Number of votes

Rank	35	28	20	17
1	N	S	J	J
2	S	N	N	S
3	J	J	S	N

Definition

A ***Condorcet winner*** in a ranked choice election is a candidate who will **win** any head-to-head election against any other candidate.

Definition

A ***Condorcet loser*** in a ranked choice election is a candidate who will **lose** any head-to-head election against any other candidate.

Condorcet Paradox

Not every election has a Condorcet winner or Condorcet loser!!

Number of votes

Rank	100	100	100
1	A	B	C
2	B	C	A
3	C	A	B

In this election, in head-to-head contests:

- A beats B
- B beats C
- C beats A

Definition

This phenomenon above is called the ***Condorcet paradox***. It is an example of a ***voting paradox***.

Condorcet winner/loser criterion

If there is a **Condorcet winner** in an election, it might be reasonable to ask that they win the election.

This is another criterion that we can ask voting systems to satisfy!

Definition

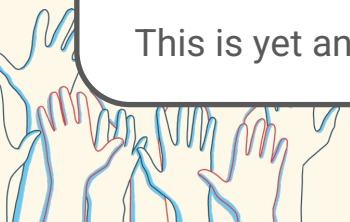
A voting system satisfies the ***Condorcet winner criterion (CWC)*** if it elects a Condorcet winner whenever one exists.

If there is a **Condorcet loser** in an election, it is reasonable to ask that they shouldn't win the election.

This is yet another criterion.

Definition

A voting system satisfies the ***Condorcet loser criterion (CLC)*** if it will *never* elect a Condorcet loser when one exists.



Condorcet winner/loser criterion

Q: How would you prove that a voting system V ***does not satisfy*** the Condorcet winner criterion?

A: Find an election where there is a Condorcet winner, but V doesn't elect them.

In this election:

- Norm is a Condorcet winner

And:

- Plurality would elect Jesse.

Definition

A voting system satisfies the ***Condorcet winner criterion (CWC)*** if it elects a Condorcet winner whenever one exists.

Number of votes

Rank	35	28	20	17
1	N	S	J	J
2	S	N	N	S
3	J	J	S	N

So **plurality does not satisfy the Condorcet winner criterion.**

Condorcet winner/loser criterion

Number of votes

Rank	35	28	20	17
1	N	S	J	J
2	S	N	N	S
3	J	J	S	N

In this election:

- Jesse is a Condorcet loser (she would lose every head-to-head contest)

And:

- Plurality would elect Jesse.

Definition

A voting system satisfies the **Condorcet loser criterion (CLC)** if it will *never* elect a Condorcet loser when one exists.

So **plurality does not satisfy the Condorcet loser criterion.**

Borda count

What about the Borda count?

Exercise: If a voting system V violates the *majority criterion* (V will elect any candidate who gets a majority of first place votes) then V will also violate the Condorcet winner criterion.

Hint: If V violates the majority criterion, this means there is an election where a candidate A got a majority of first place votes, but they weren't elected. Try to argue that A is a Condorcet winner, and therefore since they weren't elected, this means that V violates CWC.

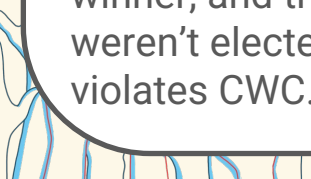
Since the **Borda count does not satisfy the majority criterion** (we saw this yesterday), this means that **the Borda count does not satisfy CWC.**

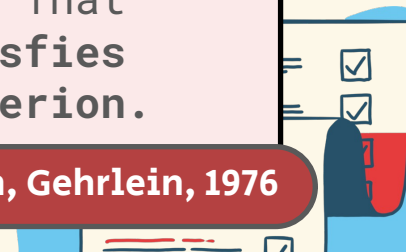
Q: Could the Borda count ever elect a Condorcet loser?

Theorem

The Borda count will never elect a Condorcet loser. That is, **the Borda count satisfies the Condorcet loser criterion.**

Fishburn, Gehrlein, 1976





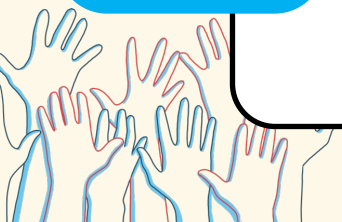
Borda count

So the status of the two big voting systems we know is as follows:

	Borda count	Plurality
Majority criterion	⊘	☑
Condorcet winner criterion	⊘	⊘
Condorcet loser criterion	☑	⊘

Discuss:

Are the Condorcet winner/loser criterion good things for voting systems to satisfy? Why or why not?



Condorcet voting

We can describe a voting system which automatically will satisfy the Condorcet winner/loser criteria.

Q: What is bad about this voting system?

A: It doesn't always give you a winner --- only when a Condorcet winner exists.

Condorcet voting

1. Everyone submits ranked *ballots* indicating who they prefer for the election
2. The ***Condorcet winner*** wins the election

We might instead want to fuse this with other voting systems, e.g.

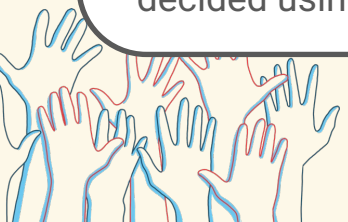
- If there is a Condorcet winner, they win
- If not, use Borda count

Instant runoff voting (IRV)

In the US, when we say “ranked choice voting,” we are generally referring to a specific type of voting system, called ***instant runoff voting***.

This is becoming increasingly adopted in city, state, and even Congressional elections.

New York’s most recent mayoral primaries were decided using IRV.



Instant runoff voting

1. Everyone submits ranked *ballots* indicating who they prefer for the election
2. Whoever got the *least number of first place votes* is eliminated, and voters’ ballots are shifted up accordingly
3. Repeat until only one candidate is left

Instant runoff voting (IRV)

Let's do an example, with the same election we used before.

Number of votes

Rank	35	28	20	17
1	N	S	J	J
2	S	N	N	S
3	J	J	S	N

Skip got the fewest first-place votes, so we eliminate them, and shift everyone's ballots up.

Number of votes

Rank	35	28	20	17
1	N		J	J
2		N	N	
3	J	J		N

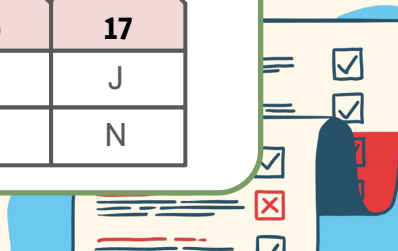
Now **Jesse** has the fewest first place votes, so we eliminate Jesse, leaving only Norm. Therefore **Norm is the winner.**

The resulting *societal preference order* is everyone in reverse-order of when they were eliminated, so:

1. Norm
2. Jesse
3. Skip

Number of votes

Rank	35	28	20	17
1	N	N	J	J
2	J	J	N	N



Instant runoff voting (IRV)

Discuss: Suppose in some stage of elimination during instant runoff voting, a candidate has a majority of first-place votes. Does this mean they win?

A: **Yes!** In each stage, a candidate can only *gain* first-place votes. They can't lose any first-place votes if they have a majority.

So if a candidate starts with a *majority of first-place votes*, then they will win.

That is, ***IRV satisfies the majority criterion*** (unlike the Borda count).

Instant runoff voting

1. Everyone submits ranked *ballots* indicating who they prefer for the election
2. Whoever got the *least number of first place votes* is eliminated, and voters' ballots are shifted up accordingly
3. Repeat until only one candidate is left

Instant runoff voting (IRV)

In the exercises we will show that

- IRV satisfies CLC
- IRV fails CWC (although there's some good evidence that this doesn't statistically happen in practice)

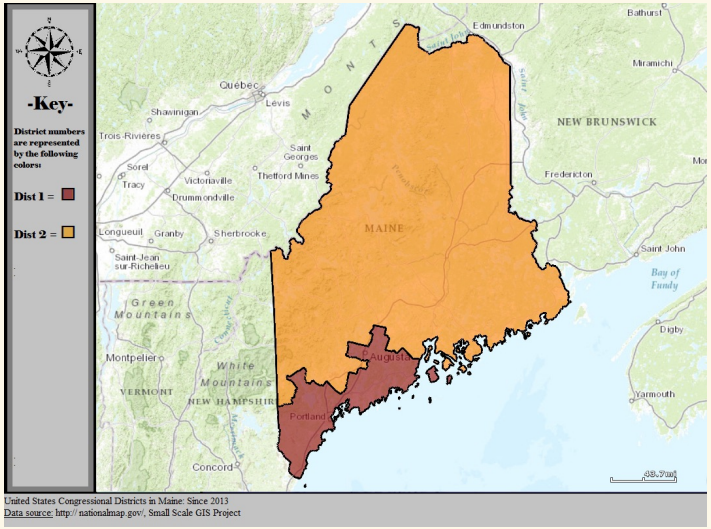
	Borda count	Plurality	IRV
Majority criterion	⊘	☑	☑
Condorcet winner criterion	⊘	⊘	⊘
Condorcet loser criterion	☑	⊘	☑

An example of IRV

The state of Maine is divided into two Congressional districts. The first and the second.

In November 2018, an election was held for the Congressperson in Maine's second district, using *instant runoff voting*.

This was the very first time that ranked choice voting had been used in a Congressional election in the United States!



The rollout of IRV in Maine was incredibly rocky, with a lot of legal challenges along the way.

Maine's 2nd district, 2018

Here are the candidates:

Bruce Polinquin, incumbent Republican, who had served since 2015

Jared Golden, Democratic challenger

Tiffany Bond, Independent, running on responsible governance, rural broadband, climate change, etc.

Will Hoar, Independent, running on ending the opioid crisis

A lot of the Bond/Hoar support was expected to come from the left. This is particularly important since the race was considered a **toss-up** between Golden and Polinquin

Maine's 2nd district, 2018

After the *ranked ballots* are submitted, the **first-choice votes** look like as follows:

U.S. House, Maine District 2 General first round, 2018 [hide]			
Party	Candidate	Vote %	Votes
Republican	Bruce Poliquin ***Incumbent***	46.33%	134,184
Democratic	Jared Golden	45.58%	132,013
Independent	Tiffany Bond	5.71%	16,552
Independent	**Will Hoar**	02.37%	6,875
Total Votes			**289,624**

Source: Maine Secretary of State, "Tabulations for Elections held in 2018," accessed January 7, 2019

We see that Polinquin got a plurality of first-choice votes. So under **plurality voting**, he would have won.

But this is instant runoff voting.

Will Hoar got the fewest first-place votes, so he is eliminated, and the ballots are shifted around.

After that, Tiffany Bond had the fewest first-place votes, so she is eliminated.

Maine's 2nd district, 2018

After the two rounds of elimination, we are left with Polinquin and Golden, and the **final votes** look like as follows:

U.S. House, Maine District 2 General first round, 2018 [hide]			
Party	Candidate	Vote %	Votes
Republican	Bruce Poliquin ***Incumbent***	46.33%	134,184
Democratic	Jared Golden	45.58%	132,013
Independent	Tiffany Bond	5.71%	16,552
Independent	**Will Hoar**	02.37%	6,875
Total Votes			**289,624**

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General election for U.S. House Maine District 2

Jared Golden defeated incumbent Bruce Poliquin, Will Hoar, and Tiffany Bond in the general election for U.S. House Maine District 2 on November 6, 2018.

		Candidate	%	Votes
✔		Jared Golden (D)	50.6	142,440
		**Bruce Poliquin** (R)	49.4	138,931

As we see, Golden gained 8,256 first-place votes after Bond/Hoar were eliminated, while Polinquin only gained 6,918. This was enough to tip the balance.

So *Jared Golden won the election under IRV*.

Maine's 2nd district, 2018

This election was a huge win for proponents of election reform. This was the first time in the US that IRV was put to use on a big stage, and it made an impact.

General election for U.S. House Maine District 2

Jared Golden defeated incumbent Bruce Poliquin, Will Hoar, and Tiffany Bond in the general election for U.S. House Maine District 2 on November 6, 2018.

	Candidate	%	Votes
✔ 	Jared Golden (D)	50.6	142,440
	Bruce Poliquin (R)	49.4	138,931

Looking at the votes, we see something interesting:

1. Only 26% of ballots actually ranked all four candidates – a lot of them only ranked a few candidates
2. 50.4% of ballots only listed one candidate!!

Discuss:

Why might voters only mark a single candidate rather than ranking all four candidates in an election like this?

Maine's 2nd district, 2018

Discuss: Why might voters only mark a single candidate rather than ranking all four candidates in an election like this?

A: It could just be a lack of information or confusion about ranked choice voting – Maine had a very rocky legal rollout of IRV.

A: It could also be the attitude of *"if my favorite candidate doesn't win then I don't want anyone else to get my vote."*

This is a form of **tactical voting**, which we will talk about in detail next week.

Key Vocab:

- Head-to-head contest
- Condorcet winner/loser
- Condorcet winner/loser criterion
- Condorcet paradox
- Instant runoff voting (IRV)

Exercises

Exercise 1: Consider an election with at least two candidates. Show that if a candidate wins this election under *majority rule*, then this candidate is a Condorcet winner.

Exercise 2: Argue that if a voting system satisfies the Condorcet winner criterion, then it will also satisfy the majority criterion.

Exercises

Exercise 3: Argue why a Condorcet loser will never win an election under instant runoff voting (that is, IRV satisfies CLC).

Exercise 4: Come up with an election that demonstrates that IRV ***fails the Condorcet winner criterion***.