2. Plurality Voting

Example

Suppose we can order one ice cream flavor for the class, and the votes come in as follows:

Flavor	Votes
Strawberry	4
Chocolate chip	3
Chocolate	2
Chocolate fudge brownie	3

Q: What flavor do we get?

A: Probably strawberry because it got the most votes

Q: Did strawberry get a *majority* of votes?

A: No but it got more votes than any other flavor. That is, it got a ***plurality*** of votes.

Plurality voting

This is probably the voting system we are most familiar with --- it is called ***plurality voting***.

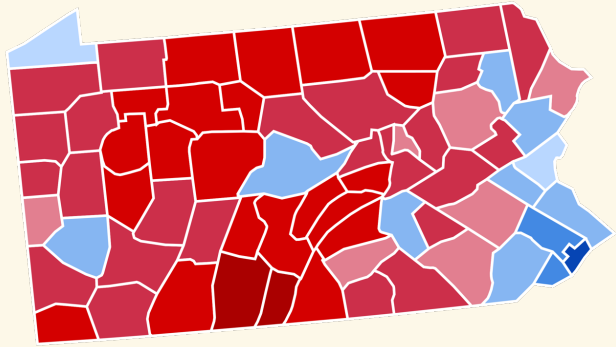
Plurality voting (for 2+ candidates)

1. Everyone submits their *ballots* indicating who they prefer for the election
2. Whoever gets a ***plurality*** of votes wins (whoever got the most votes)

Discuss:

If there are only two candidates, what is the difference between plurality and majority rules?

Pennsylvania, 2020

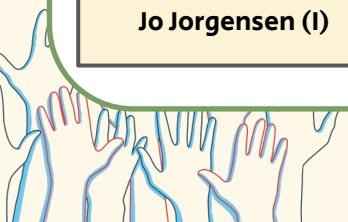


Plurality voting is how most states are decided in US elections.

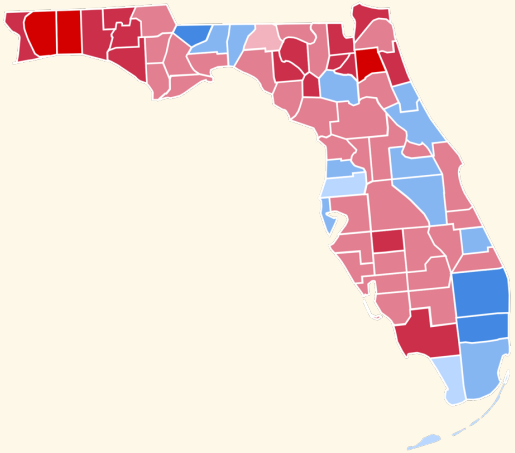
US 2020 Presidential Election, in Pennsylvania

Candidate	Votes	Percentage
Joe Biden (D)	3,458,229	50.01%
Donald Trump (R)	3,377,674	48.84%
Jo Jorgensen (I)	279,380	1.15%

Biden got a **majority** of votes, which is also a **plurality** of votes



Florida, 2000



US 2000 Presidential Election, in Florida

Candidate	Votes	Percentage
George Bush (R)	2,912,790	48.847%
Al Gore (D)	2,912,253	48.838%
Ralph Nader (I)	97,488	1.64%
Others	40,579	0.675%

No one got a **majority**.

Bush *narrowly* got a **plurality**, so he won the state

Ice cream, again

Flavor	Votes
Strawberry	4
Chocolate chip	3
Chocolate	2
Chocolate fudge brownie	3

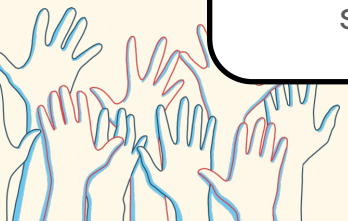
Suppose the chocolate / chocolate chip / chocolate fudge brownie voters *hate strawberry*. It's their least favorite flavor.

Discuss:

8 out of 12 voters want *anything but strawberry*. But under ***plurality***, we are ordering strawberry. Is this fair?

Discuss:

8 out of 12 voters want something chocolate-y. But we are ordering strawberry. Is this fair?



Plurality voting can start to feel unfair when we incorporate more preferences beyond people's first choices

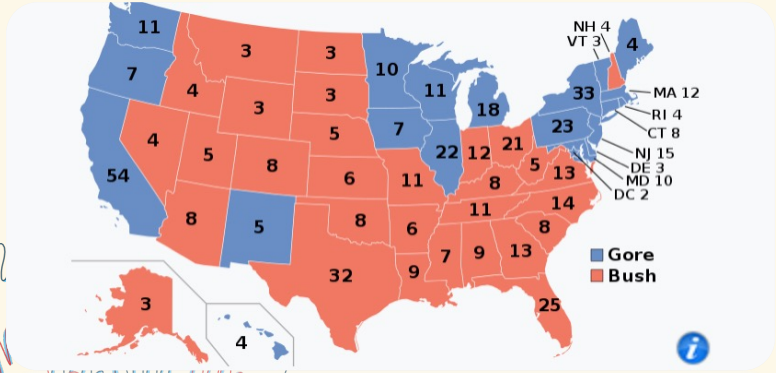
Florida, 2000,again

US 2000 Presidential Election, in Florida

Candidate	Votes	Percentage
George Bush (R)	2,912,790	48.847%
Al Gore (D)	2,912,253	48.838%
Ralph Nader (I)	97,488	1.64%
Others	40,579	0.675%

Most political scientists agree that a strong majority of Nader voters preferred Gore to Bush. Let's accept this as fact for the sake of this example. That is, if Nader had dropped out of the race at the last second, a huge block of his voters would have voted for Gore, enough to tip the balance and deliver Gore the state.

Bush won the electoral college by 5 electors (Florida had 25 at the time).



Nader had no real shot at winning the presidency, but he still affected the outcome!!

This is called the ***spoiler effect***.

Spoiler effect

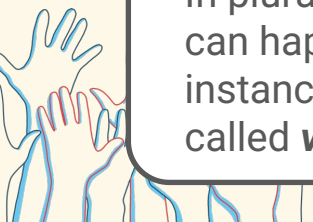
Definition

The ***spoiler effect*** is when a popular candidate loses an election because an unpopular but ideologically similar candidate attracts votes away.

Suppose for the sake of argument that all (or most) Nader voters preferred Gore to Bush. Then the following statement is true:

- A *majority* of Florida voters preferred Gore to Bush, but Bush still won.

In plurality voting, things like this can happen. The spoiler effect is an instance of a bigger phenomenon called ***vote splitting***.



Vote splitting

Definition

Vote splitting is when ideologically similar candidates split a block of votes and someone else wins.

We saw this in the ice cream example: because there were so many ”ideologically similar” chocolate options, the “chocolate” vote was split, and strawberry ended up winning.

Flavor	Votes
Strawberry	4
Chocolate chip	3
Chocolate	2
Chocolate fudge brownie	3

Duverger's law

Discuss: You are one of 10 political parties running for election in a plurality system. What strategies do you use in order to win?

A: One possible answer is to try to join forces with another party in order to avoid splitting votes.

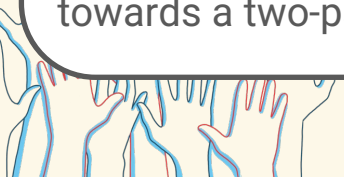
If everyone employs this strategy, we eventually get to 3 parties. The safest strategy is for 2 of them to merge in order to defeat the third.

Thus in a plurality system, strategy dictates that over time, we drift towards a two-party system.

Duverger's Law

Plurality voting favors a two-party system. Over time, a plurality voting system will eventually become two-party.

Maurice Duverger, 1964

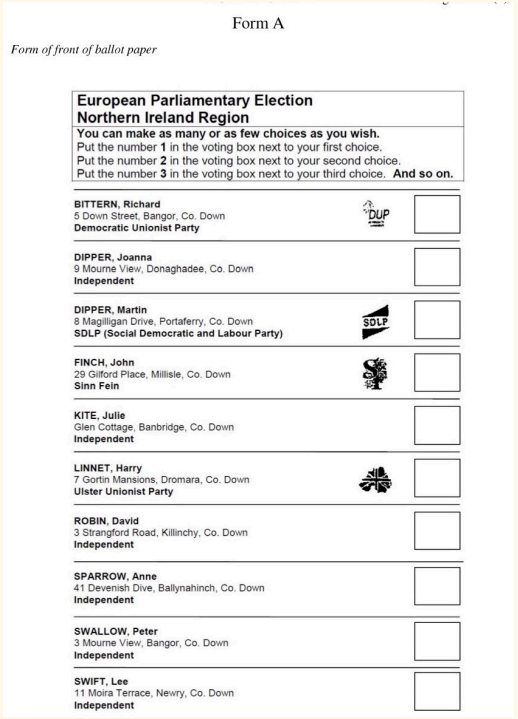


Ranked choice voting

One way to avoid some of the issues involved in plurality voting is allowing voters to submit a ballot *ranking all of the candidates*, instead of just marking their favorite.

Definition

Ranked choice voting (RCV) is any type of voting system in which voters submit a *ranking of all of the candidates* from first to last as a ballot.



Form A

Form of front of ballot paper

European Parliamentary Election
Northern Ireland Region

You can make as many or as few choices as you wish.
Put the number **1** in the voting box next to your first choice.
Put the number **2** in the voting box next to your second choice.
Put the number **3** in the voting box next to your third choice. **And so on.**

BITTERN, Richard
5 Down Street, Bangor, Co. Down
Democratic Unionist Party

DIPPER, Joanna
9 Mourne View, Donaghadee, Co. Down
Independent

DIPPER, Martin
8 Magilligan Drive, Portaferry, Co. Down
SDLP (Social Democratic and Labour Party)

FINCH, John
29 Gilford Place, Millisle, Co. Down
Sinn Fein

KITE, Julie
Glen Cottage, Banbridge, Co. Down
Independent

LINNET, Harry
7 Gortin Mansions, Dromara, Co. Down
Ulster Unionist Party

ROBIN, David
3 Strangford Road, Killinchy, Co. Down
Independent

SPARROW, Anne
41 Devenish Dive, Ballynahinch, Co. Down
Independent

SWALLOW, Peter
3 Mourne View, Bangor, Co. Down
Independent

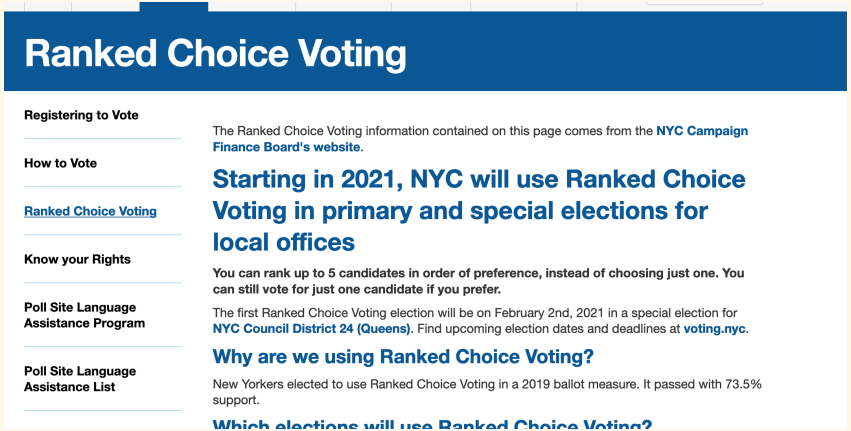
SWIFT, Lee
11 Moira Terrace, Newry, Co. Down
Independent

Example RCV ballot from Northern Ireland

We could consider plurality voting as a form of ranked choice voting, but where the voting system only looks at voters' first choices.

Ranked choice voting: examples

There are many different types of ranked choice voting. In the United States, when we say “ranked choice voting,” we are often referring to a variant called ***instant runoff voting***, which we will talk about tomorrow.



Ranked Choice Voting

Registering to Vote

How to Vote

Ranked Choice Voting

Know your Rights

Poll Site Language Assistance Program

Poll Site Language Assistance List

The Ranked Choice Voting information contained on this page comes from the **NYC Campaign Finance Board's website**.

Starting in 2021, NYC will use Ranked Choice Voting in primary and special elections for local offices

You can rank up to 5 candidates in order of preference, instead of choosing just one. You can still vote for just one candidate if you prefer.

The first Ranked Choice Voting election will be on February 2nd, 2021 in a special election for **NYC Council District 24 (Queens)**. Find upcoming election dates and deadlines at **voting.nyc**.

Why are we using Ranked Choice Voting?

New Yorkers elected to use Ranked Choice Voting in a 2019 ballot measure. It passed with 73.5% support.

Which elections will use Ranked Choice Voting?

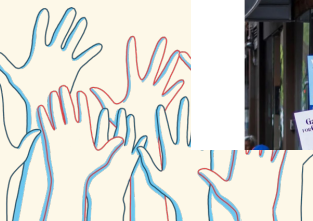
The NYC Mayoral primary in 2021 used ranked choice voting

Intelligencer

NYC MAYORAL RACE | JULY 2, 2021

The Big Winner of the NYC Mayor’s Race Was Ranked-Choice Voting

By Lee Drutman



Ranked choice voting

The MLB Most Valuable Player Award, and the Heisman Trophy in college football are both decided using ranked choice voting.

MLB

MLB MVP 2020: Jose Abreu Wins AL MVP, Freddie Freeman Captures NL Honors

TIM DANIELS

NOVEMBER 12, 2020

Ranked choice voting

Every week during the college football season, the Associated Press puts out a ranking of the top 25 college football programs in the NCAA. Here's how this works:

They reach out to reporters, journalists, etc. who each write down their own ranking of the best team (these are like *ranked choice ballots*). Then the AP decides from these ballots the ranking of the teams. This is a **voting system**.

Suppose there are three reporters, and their ballots look like this:

Rank	Reporter 1	Reporter 2	Reporter 3
1	Alabama	Alabama	Ohio State
2	Ohio State	Clemson	Clemson
3	Clemson	Ohio State	Alabama

The AP will take this, and assign a certain number of points to each ranking:

- a last place rank is worth 0 points
- a 2nd place rank is worth 1 point
- a 1st place rank is worth 2 points

Ranked choice voting

- a last place rank is worth 0 points
- a 2nd place rank is worth 1 point
- a 1st place rank is worth 2 points

Alabama gets two 1st place ranks (4 points) and no 2nd place ranks

Ohio State gets one 1st place rank (for 2 points) and one 2nd place rank (1 point) for a total of 3 points

Clemson gets two 2nd place ranks (2 points)

We call this final ranking at the end a ***societal preference order***

Rank	Reporter 1	Reporter 2	Reporter 3
1	Alabama	Alabama	Ohio State
2	Ohio State	Clemson	Clemson
3	Clemson	Ohio State	Alabama

AP's list of the top college football teams:

Rank	Team	# Points
1	Alabama	4 points
2	Ohio State	3 points
3	Clemson	2 points

Borda count

This type of voting system is called the ***Borda count***. It is named after French mathematician *Jean-Charles de Borda*, who wrote about this voting system in the 1700's

It is used for elections in the country Kiribati, and it is used to elect certain members of parliament in Slovenia.

Borda count

1. Everyone submits a ballot *ranking the candidates*
2. A last place rank is worth 0 points, second to last place is worth 1 point, and so on
3. Whoever gets the most points wins

Borda count, example

Let's do another example. Suppose Filiz, Gerald, Helen, and Ivan are running for president, and suppose there are 27 voters. Everyone submits their ballots:

How to read this: A lot of people had similar ballots. This table says that 12 people submitted a ballot ranking Filiz first, Gerald second, Helen third, and Ivan fourth.

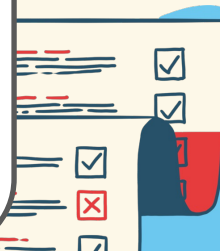
7 people submitted a ballot which ranked Gerald first, Helen second, Ivan third, and Filiz fourth… etc.

Number of votes

Rank	12	7	5	3
1	F	G	H	I
2	G	H	I	H
3	H	I	F	G
4	I	F	G	F

Since there are four candidates, we have that each place is worth the corresponding number of points:

Rank 1	3 points
Rank 2	2 points
Rank 3	1 point
Rank 4	0 points



Rank 1	3 points
Rank 2	2 points
Rank 3	1 point
Rank 4	0 points

Borda count, example

Number of votes

Rank	12	7	5	3
1	F	G	H	I
2	G	H	I	H
3	H	I	F	G
4	I	F	G	F

Filiz got 12 first place ranks, 0 second place ranks, 5 third place ranks, and we don't have to count last place ranks since they're not worth anything. This is a total of:

$12 \times 3 + 0 \times 2 + 5 \times 1 = 41$ points

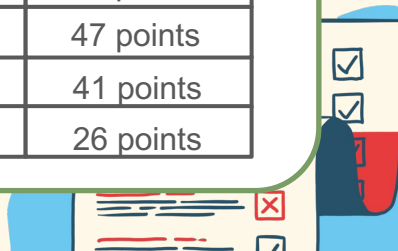
Gerald: $7 \times 3 + 12 \times 2 + 3 \times 1 = 48$ points

Helen: $5 \times 3 + 10 \times 2 + 12 \times 1 = 47$ points

Ivan: $3 \times 3 + 5 \times 2 + 7 \times 1 = 26$ points

Societal preference order

Rank	Candidate	# of points
1	Gerald	48 points
2	Helen	47 points
3	Filiz	41 points
4	Ivan	26 points



Borda count, example

Definition

The ***societal preference order*** is the "output" of a voting system, in the way that ballots are the "input". Instead of just picking a single winner, the societal preference order tells you who came in first place, second place, etc.

Societal preference order

Rank	Candidate	# of points
1	Gerald	48 points
2	Helen	47 points
3	Filiz	41 points
4	Ivan	26 points

US 2020 Presidential Election, in Pennsylvania

Candidate	Votes	Percentage
Joe Biden (D)	3,458,229	50.01%
Donald Trump (R)	3,377,674	48.84%
Jo Jorgensen (I)	279,380	1.15%

Q: What is the resulting *societal preference order* for the 2020 Pennsylvania election under ***plurality voting?***

A: It is:
1. Biden
2. Trump
3. Jorgensen

The societal preference order under the Borda count is *how many points* each candidate got.

The societal preference order under plurality is *how many votes* each candidate got.

Anonymity / unanimity

Last time, we had some criteria for voting systems with 2 candidates. Let's try to improve these definitions so that they make sense when there are 3+ candidates, and when voters *may be submitting ranked ballots*

Definition

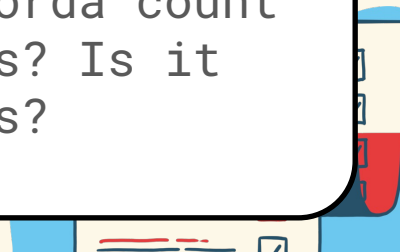
A voting system with 2+ candidates is ***anonymous*** if it treats all the voters equally. That is, if any two voters traded ballots, the outcome of the election would stay the same.

Definition

A voting system with 2+ candidates is ***unanimous*** if when ~~everyone votes for a candidate~~ ranks a candidate first, then that candidate wins.

Discuss:

Is the Borda count anonymous? Is it unanimous?



Neutrality

Our old definition of neutrality doesn't quite make sense, so we need to tweak it for elections with 3+ candidates.

Old Definition

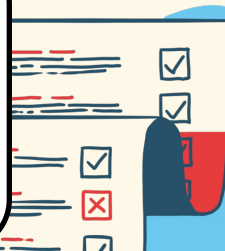
A voting system for an election with 2 candidates is *neutral* if, for any election, if candidate A wins and then everyone switches their vote, then candidate B wins.

Definition

A voting system with 2+ candidates is ***neutral*** if, for any election, and *for any two candidates A and B*, if every voter swaps the positions of A and B on their ballot, then A and B swap places in the *societal preference order*.

Q: Is the Borda count neutral?

A: Yes! If everyone swaps A and B on their ballots, then the ***number of points*** A and B got swaps, which swaps them in the societal preference order.



Q: Is plurality neutral?

A: Yes! If everyone swaps A and B on their ballots, then the ***number of votes*** A and B got swaps, which swaps them in the societal preference order.

Monotonicity

Remember the last criterion we talked about was *monotonicity*:

The essence of this is that **positive changes** for a candidate, that don't affect any other candidates, should only have **positive outcomes**. A better definition is then:

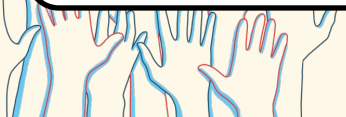
Old Definition

A voting system for an election with 2 candidates is *monotone* if it is impossible for a winning candidate to become a losing candidate by gaining a new block of votes.

Definition

A voting system with 2+ candidates is ***monotone*** if, for any candidate A, if some voters move A up in their rankings, then A will not drop down in the societal preference order.

Q: Is plurality monotone?



A: Yes! Here is what monotonicity means in plurality voting: if a winning candidate gains a new block of voters, then this won't cause them to lose.

Monotonicity

Q: Is the Borda count monotone?

A: Yes! If voters move A up in their rankings, this can only ***increase*** the number of points A gets, which can only move them higher up in the societal preference order.

Definition

A voting system with 2+ candidates is ***monotone*** if, for any candidate A, if some voters move A up in their rankings, then A will not drop down in the societal preference order.

So for our four criteria, plurality and the Borda count satisfy *all of them*.

1. This is already different than May's Theorem
2. We need *more criteria* in order to compare these voting systems

Majority criterion

Plurality satisfies the majority criterion – if a candidate gets a majority of votes (more than 50%), they automatically have a plurality, so they win.

Definition

A voting system with 2+ candidates satisfies the ***majority criterion*** if, whenever a candidate receives a majority of first-place votes, they will win.

The Borda count ***fails the majority criterion***.

To demonstrate that a voting system fails the majority criterion, we should come up with an election where a candidate won a majority of first-place votes, but that candidate loses under our voting system.

Majority criterion

Suppose 20 of us are using the **Borda count** to vote on our favorite drink, and the options are coffee, tea, and soda.

So *coffee got a majority of first-place votes*. However we check that the societal preference order under the Borda count is:

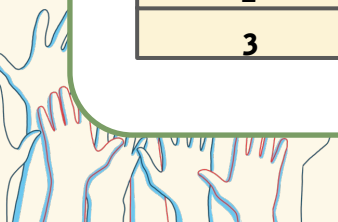
Societal preference order

Rank	Candidate	# of points
1	Tea	25 points
2	Coffee	22 points
3	Soda	11 points

Number of votes

Rank	11	7	2
1	Coffee	Tea	Soda
2	Tea	Soda	Tea
3	Soda	Coffee	Coffee

Coffee had a majority of first-place votes, but failed to win. That is, the **Borda count fails the majority criterion.**



Discuss: Is it good for a voting system to satisfy the majority criterion?

Discuss: Do you prefer the Borda count or plurality? Why?

Key Vocab:

- Majority vs. plurality
- Plurality voting
- The spoiler effect
- Vote splitting
- Duverger's law
- The Borda count
- Societal preference order
- Majority criterion

Exercises

Exercise 1: In this election:

1. Who wins under majority rule (if anyone)?
2. Who wins under plurality?
3. Who wins under the Borda count?

Rank	40	31	18	11
1	A	B	B	C
2	C	A	C	A
3	B	C	A	B

Exercise 2: Explain why, in a monotone voting system, changes that are unfavorable to a candidate cannot cause that candidate to finish higher in the societal preference order.

Exercise 3:

1. In an election with 3 candidates and 100 voters, how many first-place votes does a candidate need in order to guarantee a win under the Borda count?
2. In an election with 4 candidates and 100 voters, how many first-place votes does a candidate need in order to guarantee a win under the Borda count?
3. In an election with n candidates and 100 voters, how many first-place votes does a candidate need in order to guarantee a win under the Borda count?
4. (Hard) In an election with n candidates and k voters, how many first-place votes does a candidate need in order to guarantee a win under the Borda count?

Exercise 4: Argue that ***plurality*** is anonymous, unanimous, and satisfies the majority criterion.